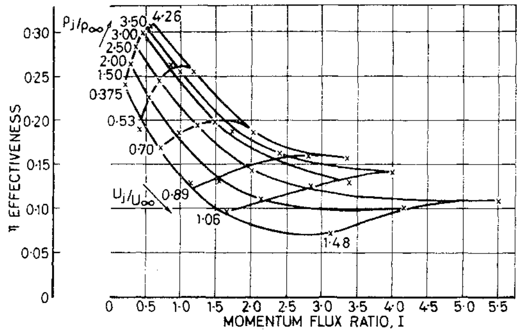

Fig. 5 Variation of effectiveness with momentum flux ratio at 6.5 diam downstream.

fectiveness close to the injection holes. This effect can also be seen in Fig. 4 where, at a fixed distance of 6.5 diam from the injection holes, the effectiveness is plotted for the full range of velocity and density ratios. Figure 5 shows that plotting the effectiveness against the momentum flux ratio I, as suggested by Goldstein,[3] does not collapse data at differing density ratios on those obtained at differing velocity ratios.

Conclusions

The results presented demonstrate that film cooling effectiveness is dependent upon the ratio of the density of the injectant to the mainstream, in addition to the velocity ratio. Further, it is apparent that account cannot be taken of the density ratio effect by working in terms of such simple parameters as mass flux ratio M or momentum flux parameter I. Until such time as a satisfactory correlating parameter has been found, it is recommended that testing of film cooling configurations be undertaken at density ratios as close as possible to those existing in practice.

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Nonexistence of Stationary Vortices Behind a Two-Dimensional Normal Plate

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THE problem is of the two-dimensional, irrotational flow of an inviscid incompressible fluid past an obstacle, with a pair of counter-rotating line-vortices lying symmetrically in the downstream flow, at rest relative to the body. Our interest in it arises from calculations we have made[1,2] of the three-dimensional flow over slender wings with leading-edge separation. If a wing has constant local span over a considerable part of its length, that is, it has a long parallel-sided region, we should expect the cross-flow in this region to be related to the two-dimensional flow of the title problem. We find that the leading-edge vortices continue to grow in strength and move upward away from the wing without apparent limit.

For the flow past a circular cylinder, Foppl showed that there is a locus of possible vortex positions, along which the circulation of the vortices varies.[3] This is what would be expected, since the vanishing of the two velocity components at the vortex positions imposes two conditions on the three unknowns, the two coordinates, and the circulation. If the obstacle is a flat plate, it is natural to impose a Kutta condition of finite velocity at the edge of the plate, and this condition enters in our calculations for slender wings. We should then expect that no more than a finite number of possible vortex positions would exist. It is surprising therefore, that both Riabouchinski[4] and Coe,[5] who introduce the Kutta condition, should again find loci for the vortex position. Our conclusion is quite different: we find no stationary vortex position behind a flat plate with the Kutta condition imposed.

It appears from a paper by Roy[6] that Villat[7] had come to the same conclusion in 1930. Since his work is not widely available and the algebra involved is not particularly lengthy, it seems worthwhile setting out the steps in full, in the hope of clarifying the situation.

We use Coe's notation in which the plate has width $2a$ and lies normal to a stream of speed U directed along the real axis of the Z-plane. Vortices of strength $\mp K$ (circulation $\mp 2\pi K$) which lie at $Z=Z_l$, and the conjugate point $\bar{Z}_l$, and the transformation $\zeta^2=Z^2+a^2$ is introduced. The vortex strength is positive for anticlockwise rotation. The Kutta condition is expressed by Coe's Eq. (4), which we write as

$$U=2K\tau/(\sigma^2+\tau^2) \tag{1}$$

by introducing σ and τ for the real and imaginary parts of $\zeta_l=(Z_l^2+a^2)^{1/2}$. The complex equation expressing the vanishing of the velocity at the position of the upper vortex is Coe's Eq. (6), which we write as

$$UZ_l-KZ_l/2\tau+iKa^2/2Z_l\zeta_l=0 \tag{2}$$

When we divide this equation by Z_l and take the imaginary part, we find

$$\Re\{Z_l^2\zeta_l\}=0$$

Also

$$Z_l^2\zeta_l=(\sigma^2-\tau^2-a^2+2i\sigma\tau)(\sigma+i\tau) \tag{3}$$

and so this condition gives

$$\sigma^2-3\tau^2=a^2 \tag{4}$$

This is the same as Coe's Eq. (7) which he describes as the locus of the vortex. His Eq. (8) does not appear to follow from his Eq. (6). We find, on dividing Eq. (2) by Z_l and now taking the real part,

$$U-K/2\tau=-Ka^2/\mathcal{I}\{2Z_l^2\zeta_l\}$$
$$=-Ka^2/4\tau(\sigma^2+\tau^2),\ \text{by (3) and (4)} \tag{5}$$

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This is the second of the two real equations that arise from Eq. (2). On the other hand, from the Kutta condition (1)

$$U-K/2\tau=-K(\sigma^2-3\tau^2)/2\tau(\sigma^2+\tau^2)$$

$$=-Ka^2/2\tau(\sigma^2+\tau^2)\text{, by (4)} \qquad (6)$$

Eqs. (5) and (6) are incompatible for finite values of σ and τ, so we conclude that no solution exists. Since Eqs. (5) and (6) differ only by a factor of 2, it seems likely that both Riabouchinsky and Coe lost this factor somewhere, leading them to think that the Kutta condition was satisfied automatically. Of course, from the equation $2=1$ anything follows, including the two different loci quoted in Refs. 4 and 5.

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Ion Density Measurements with an Electric Probe

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THIS Note reports on the application of an electric probe at large negative potential to measuring the ion density in a flowing, slightly ionized gas. The experimental conditions were such that an ion sheath formed on the ion collecting electrode. The viscous boundary layer was thin compared with the ion sheath. This allowed use of the theory of Ref. 1 for evaluation of the data. Although the theory was developed for a flat plate and the probe used was cylindrical, the theory should be quite good when the sheath is thinner than the transverse electrode dimension (i.e., for $n>10^{10}$ cm^{-3} at our conditions).

The dimensions of the probe used for these experiments are shown in Fig. 1. The Pyrex body, nominally 6 mm i.d., was hot-formed over a mandrel to provide the diverging cross section. The shape was chosen to compensate approximately for boundary-layer growth in the flow behind a Mach 9 shock into air at an initial pressure of 0.3 torr. For the conditions of interest here, the ionization relaxation time is much longer than the transit time through the probe, and hence a slight expansion or contraction of the flow (barring choking or shock formation) will not affect the ionization level of the stream. The probe leading edge was sharpened after forming. Silver paint electrodes covering 90° quadrants and an external electrostatic shield were applied by brush and fired. Fine stranded leads were soldered to the electrodes which were carried around the fire-polished back edge of the probe body so that all connections could be made on the outside surface for minimal flow disturbance. The completed probe was then coated externally with corona lacquer to insulate it from the plasma and mounted in a tapered, anodized aluminum holder. Leads were epoxied to the inside wall of the holder and brought back 12 in. to a mounting plate for connection to coaxial cable. The entire assembly was mounted on a 2-in. diam hollow rod extending 3 ft from the end of the shock tube. This avoided generation of flow disturbances or a reflected shock inside the probe during the test time.

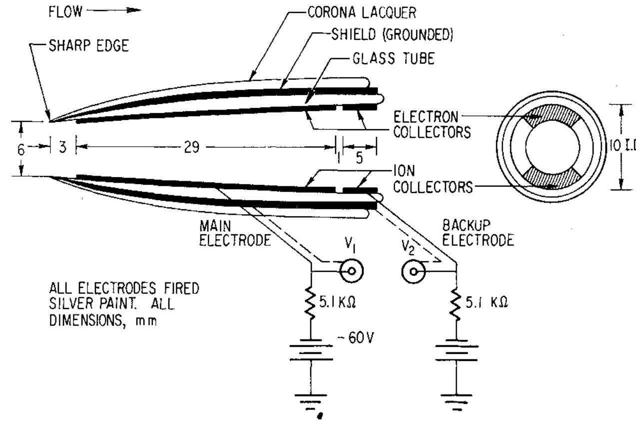

Fig. 1 Geometry of electric probe.

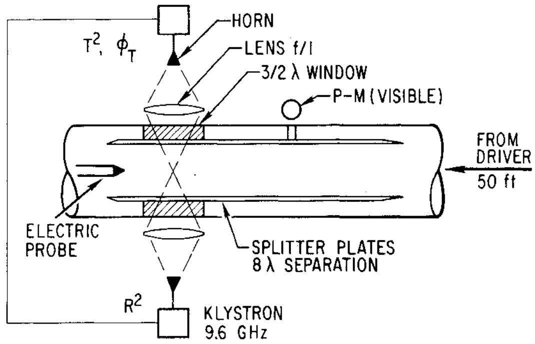

Fig. 2 Instrumentation of 17 in. shock tube.

A microwave probe[2] was used to determine the electron density independently. The overall experimental arrangement is shown in Fig. 2. The 17-in. shock tube was fitted with a rectangular box with sharpened leading edge to isolate a plane slab of gas. A focused microwave beam at 9.6 GHz ($\lambda_0=3.13$ cm) passes through resonant windows and through the gas sample. It has been shown by Primich et al.[3] that an f/1 lens system with tapered illumination (as provided by the 15 db horn pattern) has optimal resolution of approximately 3/2 wavelength between 3 db points transverse to the beam axis. It has also been shown[4,5] that the focused aperture system generates a very nearly plane wave front for a distance of $4\lambda_0$ on either side of the focus; this, and a desire for maximum sensitivity, determined the $8\lambda_0$ width of the test slab. To avoid distortion and refraction of the wave front, the focused probe should be operated in a highly underdense plasma. For all of our experiments, the collision ratio ν_c/ω was less than 0.1 and

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